Strategic retailer behaviour in an electricity contract market

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Abstract

Generators in a wholesale electricity market can exercise market power, but the existence of forward hedging contracts between consumers and generators mitigates this market power. A question arises as to why generators are willing to enter contracts with consumers to surrender their market power. In this paper, we model the role of the consumers (retailers here) in offering hedging contracts to generators. We suppose that the retailer gives the generators economic incentives to enter into contracts, quite apart from any risk premium. Our model shows that the retailer can increase its profit in comparison with the case when there are no contracts, and that the social welfare is maximized. We also consider a situation with multiple retailers and model the contracting game between the retailers.

1 Introduction

Wholesale electricity markets, in which generators compete to supply demand from retailers, are open to the exercise of market power by generators. Strategic bidding (or capacity withholding) in markets with a small number of generators can be used to enable generators to obtain higher prices than the competitive benchmark. Where there are pre-existing hedging contracts, these can act to curb generators' market power in the wholesale spot market; this effect has been discussed by Green [1999] and by other authors.

Forward contracts, futures contracts, and various options contracts are widely used to manage financial risks associated with volatile spot market demand, prices and information uncertainty in energy markets. The contracts enable generators to hedge unfavorable low spot market prices and the retailers to hedge financial losses due to high spot market prices.[1]

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[1] A number of authors have discussed the use of contracts in electricity markets from this viewpoint (see e.g.

There is another stream of research using multi-stage game theory to model the strategic interactions between the contract market and the spot market (e.g. Allaz and Vila [1993], Green [1999], Newbery [1998], Powell [1993]). In these models contracts are signed during a first stage, and then the wholesale spot market occurs during a second stage of the game. One difficulty faced by researchers is to appropriately represent the contract negotiations at the first stage. A commonly adopted assumption is that the contract price is equal to the expected spot market price. We call this *contract-spot price equivalence* and observe that it is a type of "no arbitrage" assumption.

By considering a supply function equilibrium (SFE) in the spot market, Green [1999] shows that when there is contract-spot price equivalence and the retailer is risk-neutral, then generators would not enter any contracts with retailers if the strategic interaction between generators in the contract market satisfies the Cournot conjecture; and, at the other extreme, the generators would be fully contracted if they play a Bertrand game in the contract market. Green uses contract-spot price equivalence to formulate a model without entering into all the details of the stage one contract negotiation. In a recent doctoral thesis Anderson [2004] has discussed the way that the contract and spot market interact when there is a supply function equilibrium in the contract market as well as the spot market.

The contract-spot price equivalence assumption is reasonable if market participants cannot influence the spot market price; that is, the spot market is fully competitive and the variation of market prices is only due to stochastic factors such as load variation, weather conditions, uncontrollable generating unit failure, etc. However, both theoretical models and empirical data show that generators have significant market power in the spot market leading to higher prices than the marginal generation cost Borenstein *et al.* [2002], Wolfram [1999].

In this oligopolistic market generators are aware that entering into contracts will reduce their profitability. The standard argument here is that competing generators are forced to enter contracts, since if they do not do so, but their competitor does, then they will be worse off. This is a type of prisoner's dilemma situation. The existence of a contract market results in both generators taking contract positions that end up reducing their profits. The extent to which this is a believable representation of how such markets operate has been disputed by Harvey and Hogan [2000]. These authors argue that, except where there are vesting contracts,

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Kaye *et al.* [1990], Gedra [1994] and Eydeland and Wolyniec [2003]). The foundation for research in this area has been traditional commodity pricing models (e.g. Hull [1991]).

generators are unlikely in practice to enter into such contracts due to the repeated nature of this marketplace: the repeated prisoner's dilemma typically produces a tacit collusive outcome in which, for their mutual longer term benefit, neither generator enters into a contract.

We take a somewhat different approach and question the modelling assumption that the contract price is equal to the expected spot market price, at least in the short term. When there are significant random factors in the operation of the spot market, then a difference between the two prices can be interpreted as a risk premium paid by the retailers because of their different risk profile in comparison to a generator. However in this paper we will argue more directly for a *strategic premium* in the contract market, even when market participants are risk-neutral. As we will see, once the contract-spot price equivalence assumption is laid aside, some previous 'too good to be true' results disappear (Harvey and Hogan refer to "a silver bullet that produces a surprising and unintuitive result that market power cannot coexist with the opportunity for forward trading").

It is worth considering how the operation of the contract market mechanism is modelled by other authors. Newbery [1998] assumes that generators each offer to retailers a fixed quantity in contracts at a specified price. Retailers then decide whether to accept the contracts on offer, buying either all the contracts or none (with the decision to buy the contracts being made if they are indifferent between contracting and not contracting). Newbery then argues that in a rational expectations equilibrium with risk neutral contract traders, the contract price for a base load contract must be equal to the time-weighted spot price.

This conclusion, however, may fail if the number of retailers is small. In this case an individual retailer might rationally agree to a contract with price higher than the spot market price that will occur. The retailer compares the price of the contract, and the resulting profit, with the result if the contract is not signed. But if the retailer does not sign the contract, then the generator involved has a lower contract cover and will increase their bid in the spot market. This in turn will reduce the profits to the retailer.

It may be argued that the existence of speculators in the market will force spot and contract prices to be the same (in expectation). Consider a situation in which generators offer contracts with higher prices than will occur in the spot market, but the retailers accept them on the basis that not doing so would lead to higher spot prices. Does this give an opportunity for an arbitrageur? We are concerned with financial hedging instruments here, rather than contracts for actual delivery of power. Hence the apparent arbitrage opportunity is to step

in and offer a contract to the retailer at a lower price than offered by the generator. Since this is a financial contract there is no need for the arbitrageur to have any involvement in the spot market. This is attractive to the retailer, but the action of the arbitrageur reduces the contracts sold by the generator. The result is that the spot price will go up, and the arbitrage will no longer be profitable. In some markets the precise mechanisms for arbitrage may be complex. Borenstein *et al.* [2004] discuss the behaviour of prices in the day ahead and real time markets in California from 1998 to 2000, and argue that consistent price differences were sustained for a variety of reasons including the limited number of firms able to take advantage of profitable arbitrage trades.

We will assume that the role of retailers in the spot market is passive: there is no demand side bidding and retailers do not have control over the generation decisions. In this paper, we model a situation in which the retailer can take an active role in the contract market, and give the generators economic incentives to enter into a contract.

The paper is organized as follows. Section 2 sets up our modelling environment and the supply function equilibrium model for the spot market. Section 3 establishes the retailer's contracting model and presents the main results for this paper, which show that all players can benefit from the contracts. Section 4 studies a model where the retailer chooses a single generators who will be offered a contract, and compares the market outcomes. Section 5 extends the conclusions in Section 3 to a situation in which two retailers compete, as well as discussing a Stackelberg game between the retailers in order to determine the contract quantities and prices that are offered. The appendix contains the technical proofs of lemmas and propositions.

2   Supply function equilibrium in the spot market

We assume there are two generators in the market and one retailer (later we will extend our discussion to the case with two retailers). The model can be easily extended to a situation with more than two generators. We assume perfect information in this paper. The retailer has a utility function whose marginal utility is the demand function in the spot market. The demand function, which is assumed to be known to the generators, has the following form as in Green [1999], Powell [1993]

$$A - Bp$$

where p is the market price, A and B are independent of p with A dependent on a random shock. In most electricity markets demand is nearly inelastic over a short time frame; but it is often appropriate to take an elastic demand function as here since we deal with the residual demand that the generators in question face after taking account of the supply from non-strategic price-taking generators. The two generators' cost functions are denoted by $c_1(q)$ and $c_2(q)$, respectively.

In the spot market, the two generators offer to supply $s_1(p)$ and $s_2(p)$ at a market price p. The spot market price is determined from the intersection of the demand curve and the aggregated supply curve, that is the market price p satisfies $s_1(p) + s_2(p) = A - Bp$. We use the standard argument (see Green [1999], Klemperer and Meyer [1989], Green and Newbery [1992]) that each generator maximizes its profit with respect to the market price given the residual demand as a function of price. That is, the generator sets an optimal market price on the basis of the residual demand it faces. The freedom of the generator to offer a supply function (rather than a single quantity or price) allows it to achieve an optimal response, no matter what the realisation of demand.

Given its contract position (f_i, x_i), with contract price f_i and contract volume x_i, generator i receives a payment from the contract counterparty of $x_i(f_i - p)$ if the spot market clears at price p. Thus generator i's profit at market price p is

$$\pi_i(p, x_1, x_2, f) = p(A - Bp - s_j(p) - x_i) + fx_i - c_i(A - Bp - s_j(p))$$

where $s_j(p)$ is generator j's offer with $i, j = 1, 2$ and $i \neq j$.

To find the price that maximizes generator i's profit we take the derivative of π_i to obtain the following first order optimality condition:

$$A - Bp - s_j(p) - x_i + [p - \frac{dc_i}{dq}][-B - \frac{ds_j}{dp}] = 0$$

where dc_i/dq and ds_j/dp denote the derivatives of $c_i(\cdot)$ and $s_j(\cdot)$ at $A - Bp - s_j(p)$ and p, respectively. This equation determines the price that generator i would choose once A is known (which includes the random shock). If generator i chooses to offer a supply function $s_i(p)$, given by

$$
\begin{align}
s_i(p) &= A - Bp - s_j(p) \tag{1}\\
&= x_i + (p - \frac{dc_i}{dq})(B + \frac{ds_j}{dp}), \tag{2}
\end{align}
$$

then this optimal price will be achieved for all demands.

We shall confine ourselves to the case where each generator uses a linear supply function of the form of $\alpha + \beta p$ with $\beta \geq 0$, as did Chao and Peck [1997], Green [1999] to make the models analytically tractable.[2] In addition, we assume that the two generators have quadratic cost functions, given by $c_i(q) = C_i q + 0.5 D_i q^2$, $i = 1, 2$.

We can calculate the equilibrium solution as a function of the contract quantities x_1 and x_2, with the results given in the following Proposition.[3]

Proposition 1 *The unique equilibrium solution of the form $s_i(p) = \alpha_i + \beta_i p$ has*

$$\alpha_i = (1 - D_i \beta_i) x_i - C_i \beta_i, \ i = 1, 2$$

$$\beta_i = (BW_i / 2)(\sqrt{1 + 4/(D_i BW_i)} - 1), \ i = 1, 2$$

where $W_i = (2D_i + D_1 D_2 B)/(D_1 + D_2 + D_1 D_2 B)$. The spot market price, which depends on the realisation of demand A, is

$$
\begin{aligned}
p(x_1, x_2) &= \xi_0 - \xi_1 x_1 - \xi_2 x_2, \ where \\
\xi_0 &= \frac{A + C_1 \beta_1 + C_2 \beta_2}{B + \beta_1 + \beta_2}, \\
\xi_i &= \frac{\beta_i}{(B + \beta_1 + \beta_2)(B + \beta_j)}, \quad i \neq j, i, j = 1, 2,
\end{aligned}
$$

and the quantity dispatched from generator i is

$$
\begin{aligned}
q_i(x_1, x_2) &= \psi_{i0} + \psi_{ii} x_i - \psi_{ij} x_j, \ where \\
\psi_{i0} &= \beta_i(\xi_0 - C_i) \\
\psi_{ii} &= \frac{\beta_i}{B + \beta_1 + \beta_2} \\
\psi_{ij} &= \frac{\beta_i \beta_j}{(B + \beta_1 + \beta_2)(B + \beta_i)}, \quad i \neq j, i, j = 1, 2.
\end{aligned}
$$

Proof: See Appendix.

Note that β_1, β_2 are independent of x_1, x_2 so the generators' contract positions only have an effect on the intercepts for the optimal linear supply curves, and not on their slope.[4]

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[2] Anderson and Xu [2005] discuss what happens with more general upwards-sloping supply curves.

[3] This matches the formulae derived by Green when $C_1 = C_2 = 0$.

[4] When the two generators are identical (having the same cost function, the same market beliefs and strategies), it is easy to see from these formulae for the dispatched quantities that any nonzero contract ($x_1 = x_2 > 0$) will lead to a higher generation output for each of them than in the no contract case ($x_1 = x_2 = 0$) since, when $\beta_1 = \beta_2$, $\psi_{11} - \psi_{12} = \psi_{22} - \psi_{21} > 0$.

3 The retailer's model and social optimal outcomes

We model the contracting stage by supposing that the retailer decides on a contract to offer the two generators (different contracts for each generator). Then each generator decides whether to accept the retailer's offer or reject it. We suppose that generator i already has a contract $(\bar{f}_i, \bar{Q}_i)$ with a contract price $\bar{f}_i$ and a contract volume $\bar{Q}_i$, $i = 1, 2$. These contracts may be signed at earlier stages or with different retailers.

At the first stage the retailer offers a new contract (f_i, Q_i) to generator i with contract volume Q_i at a strike price f_i, $i = 1, 2$. The generators each decide whether or not to accept the contract offer made to them. Then the generators make their offers into the spot market with parameters set out in Proposition 1. Finally demand occurs with some distribution over the random shock A.

There are four spot market outcome scenarios depending on whether the generators accept the contract offers or not. We use the notation $(0,0), (i,0), (0,j), (i,j)$ in the superscript to represent the four possible solutions according to which of generator i or j (or neither) accepts the contract offer. Thus we write $\pi_i^{(0,0)}$ for the spot market profit of generator i when neither generator has accepted the contract offer, and we write $\pi_i^{(i,0)}$ for the spot market profit of generator i when it has accepted the contract (f_i, Q_i) but generator j has not accepted the retailer's contract offer (and similarly for generator j's spot market profit). Finally $\pi_i^{(i,j)}$ is the profit for generator i when both generators accept the contracts offered.

Then the spot market price, generation quantities and generator profits when both generator i and generator j accept the contracts offered are as follows:

$$p^{(i,j)} = \xi_0 - \xi_i(\bar{Q}_i + Q_i) - \xi_j(\bar{Q}_j + Q_j),$$
$$q_i^{(i,j)} = \psi_{i0} + \psi_{ii}(\bar{Q}_i + Q_i) - \psi_{ij}(\bar{Q}_j + Q_j),$$
$$q_j^{(i,j)} = \psi_{j0} + \psi_{jj}(\bar{Q}_j + Q_j) - \psi_{ji}(\bar{Q}_i + Q_i),$$
$$\pi_i^{(i,j)} = p^{(i,j)}q_i^{(i,j)} - (p^{(i,j)} - \bar{f}_i)\bar{Q}_i - (p^{(i,j)} - f_i)Q_i - c_i(q_i^{(i,j)}),$$
$$\pi_j^{(i,j)} = p^{(i,j)}q_j^{(i,j)} - (p^{(i,j)} - \bar{f}_j)\bar{Q}_j - (p^{(i,j)} - f_j)Q_j - c_j(q_j^{(i,j)}).$$

The corresponding expressions for the other states are obtained by deleting terms containing either Q_i or Q_j (or both). All of these depend on the demand shock A.

Now we turn to the retailer's problem which is to design a contract for each of the two generators. As we mentioned before, the retailer's utility is derived from the demand function, that is, the marginal utility is the demand function. Therefore, the utility of consumption of

q units of electricity is

$$\frac{A}{B}q - \frac{0.5}{B}q^2.$$

The retailer designs two contracts (f_i, Q_i) and (f_j, Q_j) for generator i and generator j to maximize its expected benefit of consumption while giving the two generators economic incentives to enter into the contracts. The expectation here is with respect to the demand shock A. We will write Π_i for the expected value of the profit π_i for generator i (where the expectation is taken over A). So the retailer needs to choose contracts which ensure both generators are persuaded to accept the contracts offered, i.e. $\Pi_i^{(i,j)} \geq \Pi_i^{(0,0)}$ and $\Pi_j^{(i,j)} \geq \Pi_j^{(0,0)}$.

Thus we can formulate the retailer's problem as follows

$$
\begin{aligned}
\max_{(f_i, Q_i), (f_j, Q_j)} \quad & E[\tfrac{A}{B}q^{(i,j)} - \tfrac{1}{2B}(q^{(i,j)})^2 - p^{(i,j)}q^{(i,j)} \\
& + (p^{(i,j)} - \bar{f}_i)\bar{Q}_i + (p^{(i,j)} - \bar{f}_j)\bar{Q}_j + (p^{(i,j)} - f_i)Q_i + (p^{(i,j)} - f_j)Q_j] \\
\text{such that} \quad & E[p^{(i,j)}q_i^{(i,j)} - (p^{(i,j)} - \bar{f}_i)\bar{Q}_i - (p^{(i,j)} - f_i)Q_i - c_i(q_i^{(i,j)})] \geq \Pi_i^{(0,0)}, \\
& E[p^{(i,j)}q_j^{(i,j)} - (p^{(i,j)} - \bar{f}_j)\bar{Q}_j - (p^{(i,j)} - f_j)Q_j - c_j(q_j^{(i,j)})] \geq \Pi_j^{(0,0)},
\end{aligned}
\tag{3}
$$

where we have written $q^{(i,j)}$ for $q_i^{(i,j)} + q_j^{(i,j)}$.

Once the retailer has decided its offers the generators are faced with a take-it-or-leave-it game. We will show in Proposition 3 that the constraints here are enough to guarantee that the only Nash equilibrium for the take-it-or-leave-it game is one with both offers accepted. There might be a situation in which one of these constraints fails and yet there is a unique Nash equilibrium at solution (i, j). For example generator i may be worse off as a result of both generators entering into contracts, but once generator j accepts the contract offer, then generator i finds it beneficial to agree to the contract terms. Hence we might consider the more complex problem of finding the maximum retailer utility with weaker constraints, but sufficient to make the solution a unique Nash equilibrium. We will not deal with this case directly, but in the next section we will consider a related problem in which the retailer approaches just one of the generators with a contract offer. This will achieve a higher retailer profit.

Consider the choice of f_i and f_j. Once Q_i and Q_j are fixed it is easy to see that both f_i and f_j will be chosen as small as possible consistent with the two constraints being satisfied. This also can be seen by noting that the Lagrange multipliers corresponding to the two constraints at any solution of (3) must equal one. Hence we can assume that both constraints are satisfied

with equality and use these to substitute for $E[(p^{(i,j)} - \bar{f}_j)\bar{Q}_j]$ and $E[(p^{(i,j)} - f_j)Q_j]$. Since most terms cancel the problem is equivalent to

$$\max_{Q_i, Q_j} \quad E[\tfrac{A}{B} q^{(i,j)} - \tfrac{1}{2B}(q^{(i,j)})^2 - c_i(q_i) - c_j(q_j) - \pi_i^{(0,0)} - \pi_j^{(0,0)}]$$

The first order conditions for this problem are:

$$E[\tfrac{\partial}{\partial Q_i}[(A/B)q - q^2/(2B)] - \tfrac{\partial c_i(q_i)}{\partial Q_i} - \tfrac{\partial c_j(q_j)}{\partial Q_i}] = 0$$

$$E[\tfrac{\partial}{\partial Q_j}[(A/B)q - q^2/(2B)] - \tfrac{\partial c_i(q_i)}{\partial Q_j} - \tfrac{\partial c_j(q_j)}{\partial Q_j}] = 0$$

where for simplicity we have left off the superscripts $^{(i,j)}$.

Thus

$$E[\frac{A}{B} - \frac{1}{B}(q_i + q_j) - c_i'(q_i)]\frac{\partial q_i}{\partial Q_i} + E[\frac{A}{B} - \frac{1}{B}(q_i + q_j) - c_j'(q_j)]\frac{\partial q_j}{\partial Q_i} \quad = \quad 0, \qquad (4)$$

$$E[\frac{A}{B} - \frac{1}{B}(q_i + q_j) - c_i'(q_i)]\frac{\partial q_i}{\partial Q_j} + E[\frac{A}{B} - \frac{1}{B}(q_i + q_j) - c_j'(q_j)]\frac{\partial q_j}{\partial Q_j} \quad = \quad 0, \qquad (5)$$

Moreover the objective function here is concave so that a solution to the first order conditions will be a maximum (for its proof, see Proposition 7 in the Appendix). By (4) and (5), we have

$$E[\frac{A}{B} - \frac{1}{B}(q_i + q_j) - c_i'(q_i)] = 0, \text{ and } E[\frac{A}{B} - \frac{1}{B}(q_i + q_j) - c_j'(q_j)] = 0 \qquad (6)$$

since

$$\begin{pmatrix} \frac{\partial q_1}{\partial Q_1} & \frac{\partial q_2}{\partial Q_1} \\ \frac{\partial q_1}{\partial Q_2} & \frac{\partial q_2}{\partial Q_2} \end{pmatrix} = \begin{pmatrix} \frac{\beta_1}{B+\beta_1+\beta_2} & -\frac{\beta_1\beta_2}{(B+\beta_1+\beta_2)(B+\beta_2)} \\ -\frac{\beta_1\beta_2}{(B+\beta_1+\beta_2)(B+\beta_1)} & \frac{\beta_2}{B+\beta_1+\beta_2} \end{pmatrix} \qquad (7)$$

and the determinant of this matrix is $\frac{B\beta_1\beta_2}{(B+\beta_1+\beta_2)(B+\beta_1)(B+\beta_2)} > 0$.

From (1), we have

$$p - dc_i/dq_i = [q_i - (\bar{Q}_i + Q_i)]/(B + \beta_j) \qquad (8)$$

and so by (6) we can deduce that

$$E[q_i] = \bar{Q}_i + Q_i \qquad (9)$$

i.e. generator i's expected spot market generation equals its full contract volume. The same is true for generator j.

Again from (6), the retailer should set the contract volumes for each of the two generators so that the marginal generation cost of the generator in the spot market equals its marginal utility of consumption, which implies an optimal generation level in terms of social welfare.

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Moreover, the expected spot market price is equal to the marginal generation cost of each of the two generators, i.e.

$$E[p] = E[c_1'(q_1)] = E[c_2'(q_2)] \tag{10}$$

which implies that the expected social welfare is maximized at this market outcome, since the social welfare is the sum of profits of all market participants and is equal to the retailer's utility less the generation cost in our case.

From this and the fact that $E[q_1 + q_2] = \overline{A} - Bp$ (where we write $\overline{A}$ for $E[A]$) we can deduce that

$$E[q_1] = \frac{\overline{A}D_2 + C_2 - BC_1D_2 - C_1}{D_1 + D_2 + BD_1D_2}, \tag{11}$$

$$E[q_2] = \frac{\overline{A}D_1 + C_1 - BC_2D_1 - C_2}{D_1 + D_2 + BD_1D_2} \tag{12}$$

$$E[p] = \frac{\overline{A}D_1D_2 + C_1D_2 + C_2D_1}{D_1 + D_2 + BD_1D_2} \tag{13}$$

and from (9) the best choice of contract volumes are given by

$$Q_i = E[q_i] - \bar{Q}_i, \ \ Q_j = E[q_j] - \bar{Q}_j. \tag{14}$$

As we have already noted, the retailer should set the contract prices so that the two generators are better off by entering into the contracts than they would be otherwise. Thus for generator i, the contract price is

$$f_i = E[p] + \frac{\Pi_i^{(0,0)} - E[pq_i - (p - \bar{f}_i)\bar{Q}_i - c_i(q_i)]}{Q_i}, \quad i = 1, 2. \tag{15}$$

To summarize, we have established the following proposition.

Proposition 2 *The spot market output under the contract schemes in (3), where neither of the two generators has an existing over-contracted position, maximizes the expected social welfare and the expected spot market price is equal to the expected marginal generation cost of each generator. Moreover, each generator's total final contract volume is equal to its expected spot market generation.*

Now we can return to the question of whether the solution we have derived will lead to the appropriate Nash equilibrium in the game played between the two generators. We are

assuming that the retailer does not change its contract offer to one generator according to whether or not its contract offer to the other is accepted.[5]

The proposition below establishes that both generators accepting the contract offer set by the solution of (3) corresponds to a unique Nash equilibrium of the take-it-or-leave-it game for the two generators. This requires two preliminary observations on generator profits (parts (a) and (b) of the proposition).

Proposition 3   *(a) For any given demand shock A, for each generator, the attractiveness of entering a contract with the retailer is enhanced if the other generator is already contracted, i.e.for $i, j = 1, 2, i \neq j$*

$$\pi_i^{(i,j)} - \pi_i^{(0,j)} > \pi_i^{(i,0)} - \pi_i^{(0,0)};$$

(b) For any given demand shock A, for a generator that is contracted less than the amount it is dispatched in the spot market, its overall profit decreases when the other generator increases its contract cover (and increases if it is contracted more than it is dispatched);

(c) Accepting the contract offers set by (3) is the unique Nash equilibrium of the take-it-or-leave-it game for the two generators, in which each maximises its expected profit..

Proof: See Appendix.

We give an example with the demand function for the retailer and cost functions of the generators taken from [Green, 1999, p.117]. The demand function is $45 - 0.5p$ and two identical generators have a cost function of q^2. That is, $A = 45, B = 0.5, C_1 = C_2 = 0, D_1 = D_2 = 2$ in our notation. We suppose that neither of the two generator has any existing contracts initially. Table 1 lists the contract and spot market outcomes. Note that the contract price is the break-even price for the two generators. If the retailer wants to ensure that the generators will accept its contract offers, then the contract price will need to be set marginally higher.

4   Strategic contracting by the retailer

We have already mentioned that the constraints imposed on the retailer's contract offers in (3) may be unnecessarily restrictive. We can ask the question as to how the retailer should behave

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[5]As we mentioned above, Green [1999] has discussed the importance of this assumption. From the point of view of the generator we are making a Cournot assumption on conjectural variations in the contract market. Within Green's framework this would imply that generators would not choose to enter into any contracts with the retailer.

	competitive	SFE with no contract	under new contracts
market price	30	40.25	30
contract price	N/A	N/A	38.06
contract volume	N/A	0	30
total generation	30	24.87	30
retailer profit	900	618.7	658.18
generator profit	225	345.91	345.91
social welfare	1350	1310	1350

Table 1: The multi-stage game model (3) produces competitive generation outputs

in order to maximise its profit? One possibility is to choose just one of the two generators to contract with. This is equivalent to setting either Q_i or Q_j to zero. Suppose the retailer contracts with generator i , then the retailer's problem is

$$\max_{(f,Q_i)} \quad E[\tfrac{A}{B}q - \tfrac{1}{2B}q^2 - pq + (p - \bar{f}_i)\bar{Q}_i + (p - f_i)Q_i + (p - \bar{f}_j)\bar{Q}_j]$$

$$\text{such that} \quad E[pq_i - (p - \bar{f}_i)\bar{Q}_i - (p - f_i)Q_i - c_i(q_i)] \geq \Pi_i^{(0)}$$

(16)

where $\Pi_i^{(0)}$ is the expected profit of generator i when it does not accept the contract (f, Q_i) and $q = q_i + q_j$ is the total spot market generation. As before, once Q_i is fixed, f_i will be chosen to make the constraint satisfied with equality. Thus the retailer solves

$$\max_{Q_i} \quad E[\tfrac{A}{B}q - \tfrac{1}{2B}q^2 - pq_j - c_i(q_i) - \Pi_i^{(0)} + (p - \bar{f}_j)\bar{Q}_j]$$

So the first order conditions for this problem reduce to

$$E[(\frac{A}{B} - \frac{1}{B}q)\frac{\partial q}{\partial Q_i} - q_j\frac{\partial p}{\partial Q_i} - p\frac{\partial q_j}{\partial Q_i} + \bar{Q}_j\frac{\partial p}{\partial Q_i} - c_i'(q_i)\frac{\partial q_i}{\partial Q_i}] = 0.$$

The second derivative conditions show that the objective function here is concave (for its proof, see Proposition 7 in the Appendix), so a solution to the first order conditions will be a maximum. Hence the optimal contract volume Q_i satisfies

$$E[p - c_i'(q_i)]\frac{\partial q_i}{\partial Q_i} - E[q_j + \bar{Q}_j]\frac{\partial p}{\partial Q_i} = 0. \tag{17}$$

Using (8), and since $\partial q_i/\partial Q_i = \psi_{ii}$ and $\partial p/\partial Q_i = -\xi_i$, we can simplify (17) to

$$E[q_i] + E[q_j] - \bar{Q}_i - \bar{Q}_j - Q_i = 0.$$

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Thus we have established that the total contract volume for the retailer is the expected total spot market generation, i.e., the retailer is fully contracted; and the new contract volume for generator i is the expected total spot market generation less the existing contract volumes that the retailer holds.

Now from Proposition 1, we have

$$E[q_i] = E[q_i^{(0)}] + \frac{\beta_i}{B + \beta_i + \beta_j} Q_i, \quad E[q_j] = E[q_j^{(0)}] - \frac{\beta_i \beta_j}{(B + \beta_i + \beta_j)(B + \beta_j)} Q_i.$$

where $q_i^{(0)}$, $q_j^{(0)}$ are the dispatch quantities if the generator i does not accept the contract (f, Q_i). Hence the optimal contract volume is

$$Q_i = \frac{(E[q_i^{(0)}] + E[q_j^{(0)}] - \bar{Q}_i - \bar{Q}_j)(B + \beta_i + \beta_j)(B + \beta_j)}{(B + \beta_j)^2 + \beta_i \beta_j},$$

and the contract price is given by

$$f = E[p] + \frac{\Pi_i^{(0)} - E[pq_i - (p - \bar{f}_i)\bar{Q}_i - c_i(q_i)]}{Q_i}$$

where the spot market price p is given in Proposition 1 with $x_i = \bar{Q}_i + Q_i$ and $x_j = \bar{Q}_j$.

In the following example of Table 2, the retailer has a demand function $A - Bp = 45 - 0.5p$ and generator 1 and generator 2 have cost functions of $q + q^2$ and $4q + 1.25q^2$ respectively. Neither of them has a pre-existing contract. In the table, C stands for the fully competitive case, EC stands for the case with contracting prices and volumes set by (15) and (14) respectively, G1 = contracting generator 1 only, G2 = contracting generator 2 only. In the contract item fields, the first component of the pair of numbers is the contract price and the second is the contract volume.

We can see that the retailer makes a greater profit from choosing to contract with just one generator, with it being preferable to contract with generator 1 (which has lower costs). In this arrangement the generator who is not offered a contract does poorly and the overall social welfare is below that achieved in the absence of any contracts.

5   Competition between retailers

In this section we consider a situation with competition for contracts between different retailers. Suppose that there are two retailers: retailer 1 has a demand function $A_1 - B_1 p$ and retailer 2 has a demand function $A_2 - B_2 p$. There are different ways in which the competition

	C	SFE	EC	G1	G2
spot market price	33.64	43.35	33.64	32.8	34.76
total generation	28.18	23.32	28.18	28.6	27.6
contract (gen 1)	N/A	N/A	(40.52, 16.32)	(38.08, 28.6)	(0,0)
contract (gen 2)	N/A	N/A	(42.05, 11.86)	(0,0)	(39.06, 27.62)
gen 1's generation	16.32	12.82	16.32	20.91	10.2
gen 2's generation	11.86	10.5	11.86	7.69	17.4
gen 1's profit	266.39	378.57	378.57	378.57	240.62
gen 2's profit	175.74	275.45	275.45	147.53	275.45
retailer profit	794.03	544.02	582.14	667.08	644.17
social welfare	1236.16	1198.04	1236.16	1193.19	1160.23

Table 2: Effects of the retailer's contracting strategy on market outcomes

between the retailers might take place. To begin with we suppose that the retailers offer the two generators contracts simultaneously. We continue to assume that generators will not enter into contracts if their profits are reduced due to the contract agreements with the two generators. Throughout this section we will assume that the two retailers know the generation costs of the two generators and their demand functions.

Denote the contracts offered by retailer 1 to generators i, j by (f_i, Q_i) and (f_j, Q_j) respectively and the contracts from retailer 2 to the two generators by (g_i, R_i) and (g_j, R_j) respectively (where the first component is the contract price and the second is the contract volume as before).

We look for a Nash equilibrium in the contract game played between the two retailers. If a Nash equilibrium exists then the contracts are determined by the following utility maximization problems:

$$
\begin{aligned}
\max_{(f_i, Q_i), (f_j, Q_j)} \quad & E[\tfrac{A_1}{B_1} u - \tfrac{1}{2B_1} u^2 - p\, u + (p - f_i)Q_i + (p - f_j)Q_j] \\
\text{such that} \quad & E[pq_i - (p - f_i)Q_i - (p - g_i)R_i - c_i(q_i)] \geq \Pi_i^{(0,0)}, \\
& E[pq_j - (p - f_j)Q_j - (p - g_j)R_j - c_j(q_j)] \geq \Pi_j^{(0,0)}
\end{aligned}
\tag{18}
$$

and

$$\max_{(g_i,R_i),(g_j,R_j)} \quad E[\tfrac{A_2}{B_2}v - \tfrac{1}{2B_2}v^2 - pv + (p - g_i)R_i + (p - g_j)R_j]$$

$$\text{such that} \quad E[pq_i - (p - f_i)Q_i - (p - g_i)R_i - c_i(q_i)] \geq \Pi_i^{(0,0)}, \tag{19}$$

$$E[pq_j - (p - f_j)Q_j - (p - g_j)R_j - c_j(q_j)] \geq \Pi_j^{(0,0)}$$

where $\Pi_i^{(0,0)}, \Pi_j^{(0,0)}$ are the expected spot market profit of generator i and generator j when neither generator accepts the contract offers; $u = A_1 - B_1 p$ and $v = A_2 - B_2 p$ are the consumptions of retailer 1 and retailer 2, respectively; and the spot price p and the generation quantities q_1, q_2 when the contract offers are accepted are as given in Section 2 with $A = A_1 + A_2$ and $B = B_1 + B_2$.

In the same way as the derivation in Section 3, we can obtain the first order optimality conditions with respect to the contract volumes of (18) and (19). Using the identity $q_1 + q_2 - u = v$, we can show

$$\begin{pmatrix} \frac{\partial q_1}{\partial Q_1} & \frac{\partial q_2}{\partial Q_1} \\ \frac{\partial q_1}{\partial Q_2} & \frac{\partial q_2}{\partial Q_2} \end{pmatrix} \begin{pmatrix} E[p - c_1'(q_1)] \\ E[p - c_2'(q_2)] \end{pmatrix} = \begin{pmatrix} E[R_1 + R_2 - v]\frac{\partial p}{\partial Q_1} \\ E[R_1 + R_2 - v]\frac{\partial p}{\partial Q_2} \end{pmatrix} \tag{20}$$

and

$$\begin{pmatrix} \frac{\partial q_1}{\partial R_1} & \frac{\partial q_2}{\partial R_1} \\ \frac{\partial q_1}{\partial R_2} & \frac{\partial q_2}{\partial R_2} \end{pmatrix} \begin{pmatrix} E[p - c_1'(q_1)] \\ E[p - c_2'(q_2)] \end{pmatrix} = \begin{pmatrix} E[Q_1 + Q_2 - u]\frac{\partial p}{\partial R_1} \\ E[Q_1 + Q_2 - u]\frac{\partial p}{\partial R_2} \end{pmatrix}. \tag{21}$$

Now the two matrices on the left hand side are equal (and given by (7)) and $\partial p/\partial Q_1 = \partial p/\partial R_1$, and so with optimal contracts the expected uncontracted exposure for the two retailers will be equal, that is $Q_i + Q_j - E[u] = R_i + R_j - E[v]$.

Lemma 4 *At the equilibrium point the two retailers are fully contracted, that is $E[u] = Q_1 + Q_2$ and $E[v] = R_1 + R_2$.*

Proof: See Appendix.

From this lemma and (20) we have

$$E[p] = E[c_1'(q_1)] \quad \text{and} \quad E[p] = E[c_2'(q_2)],$$

that is, the expected spot market price is equal to the expected marginal cost of generation. Consequently the expected social welfare is maximized at this equilibrium.

The equation $E[p] = E[c_i'(q_i)]$ also implies that each generator is contracted the same amount as it expects to be dispatched, because $E[p - c_i'(q_i)] = (E[q_i] - (R_i + Q_i))/(B + \beta_j)$ for $i = 1, 2$.

Now we show that the solutions to (18) and (19) are Nash equilibria of the game (18) and (19) by showing that the Hessian of the Lagrangian function of each of (18) and (19) is negative semi-definite. We only show this for (18) since the other is similar.

We need to be careful about the choices of the prices f_1 and f_2, so rather than eliminate them we retain the constraints and form the Lagrangian. The KKT conditions with respect to f_1 and f_2 then imply that the unique Lagrange multipliers for both of the constraints are 1 (we do not deal with the cases where either $Q_i = 0$ or $Q_j = 0$ or $Q_i = Q_j = 0$ since the analysis for them are the same).

Given the Lagrange multipliers, the Lagrangian of the problem (18) is

$$L(f_i, f_j, Q_i, Q_j) = \quad E[\frac{A_1}{B_1}u - \frac{1}{2B_1}u^2 - p\,u + p(q_i + q_j) - c_i(q_i) - c_j(q_j)$$
$$-(p - g_i)R_i - (p - g_j)R_j - \pi_i^{(0,0)} - \pi_j^{(0,0)}]$$

which is independent of f_i, f_j. From Proposition 7 we know that L is a concave function of Q_i, Q_j. So any solution to the first order conditions of (18) or (19) is a maximizer of the problem. Hence, any solution to the joint system of the first order conditions of (18) and (19) is a Nash equilibrium of the game.

Therefore, based on the analysis above, we have the following proposition.

Proposition 5 *At a Nash equilibria for (18) and (19) between the two retailers:*

(a) the expected spot market price is equal to the expected marginal cost of generation;

(b) the expected generation of each generator is equal to its total contract volume;

(c) the expected consumption of each retailer is equal to its total contract volume.

Moreover any solution which satisfies (b) and (c) and has contract prices that make the inequalities of (18) satisfied with equality, is a Nash equilibrium.

We can calculate the expected total generation levels for each generator; just as before they are given by the expressions (11)-(12). So this determines $Q_1 + R_1$ and $Q_2 + R_2$. Moreover the consumption of each retailer is determined from the spot price level which fixes $Q_1 + Q_2$ and $R_1 + R_2$, the total contract volumes for the two generators. However there is still one degree of freedom, and so the individual contract quantities are not determined.

Moreover, even if we fix the contract quantities, we will not be able to determine the contract prices from the fact that the constraints in (18) and (19) are binding, unless we add

a further restriction, such as asking that the prices offered to different generators by a single retailer are the same. The indeterminancy in contract prices is more significant than that in contract quantities, since it affects the profits for the two retailers.

The difficulty of coordinating on a single equilibrium if contracts are offered simultaneously makes it interesting to consider an alternative possible mechanism in the contract market. We suppose that retailer 1 acts as the leader and announces its contract volumes and one single contract price for the two generators and retailer 2 observes the contract positions of retailer 1 and offers its own contracts to the two generators. Further we suppose that if the contract prices needed from retailer 2 to compensate the two generators are too high (so that its profit is worse than by not entering any contracts), then there will be no contracts between the retailers and generators. Therefore, retailer 1 needs to give an incentive to retailer 2 to sign contracts with the two generators. We can represent the resulting problem as follows:

$$\max_{f,Q_i,Q_j} \quad E[\frac{A_1}{B_1}u - \frac{1}{2B_1}u^2 - pu + (p-f)(Q_i + Q_j)]$$

$$\text{such that} \begin{cases} E[\frac{1}{2B_2}v^2 + (p-g_i)R_i + (p-g_j)R_j] \geq \frac{(E[A_2]-B_2\xi_0)^2}{2B_2}, \\ \text{and } p, g_i, g_j, R_i, R_j \text{ are given by the solution to:} \\ \quad \max_{(g_i,R_i),(g_j,R_j)} \quad E[\frac{A_2}{B_2}v - \frac{1}{2B_2}v^2 - pv + (p-g_i)R_i + (p-g_j)R_j] \\ \quad \text{such that} \quad E[pq_i - (p-f)Q_i - (p-g_i)R_i - c_i(q_i)] \geq \Pi_i^{(0)}, \\ \qquad\qquad\qquad E[pq_j - (p-f)Q_j - (p-g_j)R_j - c_j(q_j)] \geq \Pi_j^{(0)}, \end{cases} \quad (22)$$

As before $u = A_1 - B_1 p$ and $v = A_2 - B_2 p$ are the consumptions of retailer 1 and retailer 2; p, q_i and q_j are the market price and generation quantities when all contracts are accepted; and $\Pi_i^{(0)}, \Pi_j^{(0)}$ are expected generator profits when contracts are not accepted. We have slightly simplified the formulation by observing that $v^2/(2B_2)$ is the surplus of retailer 2 from consuming v units of electricity, since $A_2 v/B_2 - v^2/(2B_2) - pv = v^2/(2B_2)$.

In this leader-follower environment we have the following proposition.

Proposition 6 *If the contracting takes place according to (22) then:*

1) *the expected spot market price is equal to the expected marginal cost of generation which implies that the social welfare is maximized;*

2) *the expected consumption of each of the two retailers is equal to its total contract volume;*

3) *the expected generation of each of the two generators is equal to its total contract volume.*

17

Proof: See Appendix.

From this result and Proposition 5 we see that a solution to this system will also be a solution to (18) and (19). We can show that the contract price offered by the leader (retailer 1) is unique. However, the other quantities such as Q_i, Q_j may not be unique. Therefore, there will be different optimal contract offers by the follower (retailer 2) uniquely depending on Q_i, Q_j. The profits and the total contract volume for each of the retailers and generators, and spot market price are unique, i.e., independent of the optimal contract offers.

We conclude this section by giving a numerical example in Table 3 to the above model. In this example, we have two generators with their cost functions as given in Section 3 and two retailers with symmetric demand functions $A_1 = A_2 = A/2 = 22.5, B_1 = B_2 = B/2 = 0.25$ where A, B are given in Section 3.

We implement the model (22) in the General Algebraic Modeling System (GAMS).[6] The contracts offered by retailer 1 are: $(36.75, 7.56)$ for generator 1 and $(36.75, 7.44)$ for generator 2; contracts offered by retailer 2: $(39.39, 7.44)$ for generator 1 and $(39.35, 7.56)$ for generator 2. Recall that all the first components of the contracts are the contract prices and the second are the contract volumes. The consumptions by the two generators are equal, 15 units each for the fully competitive and contracted cases and 12.44 units for the no contract case. We can see that in this leader-follower arrangement the full additional retailer benefit (in comparison with the supply function equilibrium without contracts) flows to retailer 1.

6   Conclusions

Contracts are traditionally used to manage market participants' financial risks arising from the random variation in market prices and uncertainties in the production processes. In this paper we have examined a wholesale electricity contract market in which retailers set the terms of the contracts, but need to make these attractive to the generators. With this model, it is natural to assume that there is a contract premium even when the market participants are risk neutral. We have shown that contracts can play a strategic role in determining market

---

[6]We reformulate the problem as an ordinary nonlinear programming problem by replacing the follower's problem with its first order optimality conditions. We take advantage of the strict complementarity conditions between the two constraints and their Lagrange multiplers (In general, this will lead to a mathematical program with complemenarity constraints, see Luo *et al.* [1996]). So, in our formulation, these two constraints become equality constraints and the conditions corresponding to the derivatives with respect to R_i, R_j are (21). So we can call nonlinear programming solvers in GAMS to solve the reformulated problem.

	competitive	SFE with no contract	under contracts of (22)
market price	30	40.25	30
total generation	30	24.88	30
retailer 1 profit	450	309.39	348.79
retailer 2 profit	450	309.39	309.39
generator 1 profit	225	345.91	345.91
generator 1 profit	225	345.91	345.91
social welfare	1350	1310.6	1350

Table 3: Outcomes of contracting procedures under (22)

outcomes, since there is a benefit to the retailers in offering contracts. The results in Section 3 and Section 5 show that, under a variety of mechanisms, allowing strategic contract bids by retailers will give spot market outcomes where the marginal cost of generation is equal to the marginal utility of consumption, and hence social welfare is maximized.

7 Appendix

Proof of Proposition 1

The optimal linear supply function offered by generator i satisfies

$$
\begin{aligned}
\alpha_i + \beta_i p \quad &= s_i(p) = x_i + [p - C_i - D_i(\alpha_i + \beta_i p)](B + \beta_j) \\
&= x_i - (C_i + D_i\alpha_i)(B + \beta_j) + (1 - D_i\beta_i)(B + \beta_j)p
\end{aligned}
$$

for any p. Therefore, the coefficients $(\alpha_1, \beta_1, \alpha_2, \beta_2)$ of the equilibrium supply curves are a solution to the following quadratic system:

$$
\begin{aligned}
&\alpha_1(1 + D_1 B) + C_1\beta_2 + D_1\alpha_1\beta_2 = x_1 - C_1 B, \\
&\beta_1(1 + D_1 B) - \beta_2 + D_1\beta_1\beta_2 = B, \\
&\alpha_2(1 + D_2 B) + C_2\beta_1 + D_2\alpha_2\beta_1 = x_2 - C_2 B, \\
&\beta_2(1 + D_2 B) - \beta_1 + D_2\beta_2\beta_1 = B.
\end{aligned}
\tag{23}
$$

The unique upward sloping solution to the above system is as given in the proposition statement.

Given these supply offers, the spot market price, which depends on the demand A, is

$$
\begin{aligned}
p(x_1, x_2) &= \frac{A - \alpha_1 - \alpha_2}{B + \beta_1 + \beta_2} \\
&= \frac{A + C_1\beta_1 + C_2\beta_2 - (1 - D_1\beta_1)x_1 - (1 - D_2\beta_2)x_2}{B + \beta_1 + \beta_2} \\
&= \frac{A + C_1\beta_1 + C_2\beta_2}{B + \beta_1 + \beta_2} - \frac{1 - D_1\beta_1}{B + \beta_1 + \beta_2}x_1 - \frac{1 - D_2\beta_2}{B + \beta_1 + \beta_2}x_2.
\end{aligned}
$$

This has the form we require, since $(1 - D_i\beta_i)(B + \beta_j) = \beta_i$.

The first generator is dispatched a quantity

$$
\begin{aligned}
q_1(x_1, x_2) &= \alpha_1 + \beta_1 p \\
&= \beta_1\xi_0 - C_1\beta_1 + (1 - D_1\beta_1 - \beta_1\xi_1)x_1 - \beta_1\xi_2 x_2 \\
&= \beta_1(\xi_0 - C_1) + \frac{\beta_1}{B + \beta_1 + \beta_2}x_1 - \beta_1\xi_2 x_2
\end{aligned}
$$

which gives the coefficients we claim. A similar argument applies for the second generator. $\blacksquare$

Proof of Proposition 3

(a) From the expressions for the profit under different scenarios we have

$$
\begin{aligned}
\pi_i^{(i,0)} - \pi_i^{(0,0)} &= p^{(i,0)}(q_i^{(i,0)} - \bar{Q}_i - Q_i) - p^{(0,0)}(q_i^{(0,0)} - \bar{Q}_i) + f_iQ_i - c_i(q_i^{(i,0)}) + c_i(q_i^{(0,0)}) \\
&= (p^{(0,0)} - \xi_iQ_i)(q_i^{(i,0)} - \bar{Q}_i - Q_i) - p^{(0,0)}(q_i^{(0,0)} - \bar{Q}_i) + f_iQ_i - c_i(q_i^{(i,0)}) + c_i(q_i^{(0,0)}) \\
&= p^{(0,0)}Q_i(\psi_{ii} - 1) - \xi_iQ_i(q_i^{(i,0)} - \bar{Q}_i - Q_i) + f_iQ_i - c_i(q_i^{(i,0)}) + c_i(q_i^{(0,0)}).
\end{aligned}
$$

Similarly:

$$
\pi_i^{(i,j)} - \pi_i^{(0,j)} = p^{(0,j)}Q_i(\psi_{ii} - 1) - \xi_iQ_i(q_i^{(i,j)} - \bar{Q}_i - Q_i) + f_iQ_i - c_i(q_i^{(i,j)}) + c_i(q_i^{(0,j)}).
$$

Hence

$$
\begin{aligned}
&(\pi_i^{(i,j)} - \pi_i^{(0,j)}) - (\pi_i^{(i,0)} - \pi_i^{(0,0)}) \\
= \ &(p^{(0,j)} - p^{(0,0)})Q_i(\psi_{ii} - 1) - \xi_iQ_i(q_i^{(i,j)} - q_i^{(i,0)}) - c_i(q_i^{(i,j)}) + c_i(q_i^{(0,j)}) + c_i(q_i^{(i,0)}) - c_i(q_i^{(0,0)}) \\
> \ &-\xi_jQ_jQ_i(\psi_{ii} - 1) + \xi_iQ_iQ_j\psi_{ij}
\end{aligned}
$$

using the fact that $c(x) + c(x + a - b) < c(x - b) + c(x + a)$ if $a > 0$, $b > 0$ (since c is convex $(a + b)c(x) < ac(x - b) + bc(x + a)$ and $(a + b)c(x + a - b) < bc(x - b) + ac(x + a)$).

Now, as $\psi_{ii} < 1$ and all other quantities are positive, we can deduce that

$$
\pi_i^{(i,j)} - \pi_i^{(0,j)} > \pi_i^{(i,0)} - \pi_i^{(0,0)}.
$$

20

(b) Assume that a contract of size Q_i has been accepted by generator i. Writing p, q_i, π_i for the spot market price, dispatch quantity and profit of generator i in the spot market, we have

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial Q_j} &= \frac{\partial p}{\partial Q_j}[q_i - (\bar{Q}_i + Q_i)] + (p - \frac{dc_i}{dq_i})\frac{\partial q_i}{\partial Q_j} \\
&= [\frac{\partial p}{\partial Q_j} + \frac{1}{B+\beta_j}\frac{\partial q_i}{\partial Q_j}][q_i - (\bar{Q}_i + Q_i)] \\
&= -\frac{\beta_j}{(B+\beta_i)(B+\beta_j)}[q_i - (\bar{Q}_i + Q_i)] \\
&\begin{cases} < 0, & \text{if } q_i > \bar{Q}_i + Q_i \\ \geq 0, & \text{if } q_i \leq \bar{Q}_i + Q_i \end{cases}
\end{aligned}
$$

since $p - dc_i/dq_i = [q_i - (\bar{Q}_i + Q_i)]/(B + \beta_j)$ by (1).

(c) Begin by fixing the random shock A. We will use part (b) to establish that $\pi_i^{(i,0)} > \pi_i^{(i,j)}$. Consider the change in generator i profit as Q_j is increased from zero. In order for $\partial \pi_i/\partial Q_j$ to remain negative, we need to show that the dispatch quantity, q_i, remains greater than $\bar{Q}_i + Q_i$ over the range of values for Q_j. But notice that the quantity q_i is a decreasing function of Q_j. Thus $q_i \geq q_i^{(i,j)}$ which is $\bar{Q}_i + Q_i$ from (9).

Since the solution to the retailer's problem is constrained to have $\pi_i^{(i,j)} \geq \pi_i^{(0,0)}$, we can deduce that $\pi_i^{(i,0)} > \pi_i^{(0,0)}$. So the solution in which neither generator accepts the contract offer is not a Nash equilibrium. Moreover, using part (a), $\pi_i^{(i,j)} > \pi_i^{(0,j)}$ and from a similar argument for generator j, $\pi_j^{(i,j)} > \pi_j^{(i,0)}$. Thus the solution in which each generator accepts the contract offer is a Nash equilibrium, and is unique. Since this holds for any realisation of the random shock A it also holds in expectation. ∎

Proof of lemma 4. First, we have

$$
\begin{aligned}
R_1 + R_2 - v &= R_1 + R_2 - (q_1 + q_2 - u) \\
&= (R_1 + Q_1 - q_1) + (R_2 + Q_2 - q_2) + u - Q_1 - Q_2 \\
&= -(B + \beta_2)(p - c_1'(q_1)) - (B + \beta_1)(p - c_2'(q_2)) + u - Q_1 - Q_2.
\end{aligned}
$$

Let $\delta = Q_1 + Q_2 - E[u]$ be the expected value of the uncontracted exposure. So

$$
(B + \beta_2)E[p - c_1'(q_1)] + (B + \beta_1)E[p - c_2'(q_2)] = -2\delta.
$$

But from the linear system (21), we can show that

$$
E[p - c_1'(q_1)] = \delta\frac{(B + \beta_1)(B + \beta_2)}{B\beta_1\beta_2}(\beta_2\frac{\partial p}{\partial Q_1} + \frac{\beta_1\beta_2}{B + \beta_2}\frac{\partial p}{\partial Q_2})
$$

and

$$E[p - c'_2(q_2)] = \delta \frac{(B + \beta_1)(B + \beta_2)}{B\beta_1\beta_2} \left(\frac{\beta_1\beta_2}{B + \beta_1} \frac{\partial p}{\partial Q_1} + \beta_1 \frac{\partial p}{\partial Q_2} \right).$$

Since $\partial p / \partial R_1 = -\xi_1, \partial p / \partial R_2 = -\xi_2$, we have

$$
\begin{aligned}
-2\delta \quad &= (B + \beta_2)E[p - c'_1(q_1)] + (B + \beta_1)E[p - c'_2(q_2)] \\
&= -\delta \tfrac{(B+\beta_1)(B+\beta_2)}{B\beta_1\beta_2}(B + \beta_1 + \beta_2)(\beta_2\xi_1 + \beta_1\xi_2) \\
&= -2\delta - \delta \tfrac{\beta_1+\beta_2}{B}.
\end{aligned}
$$

after some algebra. Therefore $\delta = 0$. ∎

Proof of Proposition 6. We considering retailer 2's profit maximization problem. The first order optimality conditions are derived in the same way as before, so that the conditions (21) hold and the constraints are binding. Hence

$$
\begin{aligned}
E[pq_i - (p - f)Q_i - (p - g_i)R_i - c_i(q_i)] &= \Pi_i^{(0)}, \\
E[pq_j - (p - f)Q_j - (p - g_j)R_j - c_j(q_j)] &= \Pi_j^{(0)},
\end{aligned}
$$

Now we can reformulate the problem for retailer 1 as

$$\max_{f, Q_i, Q_j} \quad E[\tfrac{A_1}{B_1}u - \tfrac{1}{2B_1}u^2 - p\,u + (p - f)(Q_i + Q_j)]$$

$$\text{such that} \quad
\begin{aligned}
&E[\tfrac{1}{2B_2}v^2 + pq_i - (p - f)Q_i - c_i(q_i)] - \Pi_i^{(0)} \\
&\quad + E[pq_j - (p - f)Q_j - c_j(q_j)] - \Pi_j^{(0)} \geq \tfrac{(E[A_2] - B_2\xi_0)^2}{2B_2},
\end{aligned}$$

recognizing that p, q_i, q_j depend on the values of R_i, R_j which arise from solving retailer 2's problem. In fact, from the results in Section 3, we know that R_i, R_j, g_i, g_j uniquely depend on f, Q_i, Q_j and this dependence is continuous differentiable if $R_i > 0, R_j > 0$ or more generally the strict complementarity conditions hold between the constraints and corresponding Lagrange multipliers (see Fiacco [1983] for example), since the strong second order sufficient conditions hold at such solutions.

The optimality conditions for retailer 1's problem turn out to be

$$
\begin{pmatrix} \frac{\partial q_1}{\partial Q_1} & \frac{\partial q_2}{\partial Q_1} \\ \frac{\partial q_1}{\partial Q_2} & \frac{\partial q_2}{\partial Q_2} \end{pmatrix}
\begin{pmatrix} E[p - c'_1(q_1)] \\ E[p - c'_2(q_2)] \end{pmatrix}
= \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\tag{24}
$$

and

$$E[\frac{v^2}{2B_2} + p(q_i + q_j) - (p - f)(Q_i + Q_j) - c_j(q_j) - c_i(q_i)] - \Pi_i^{(0)} - \Pi_j^{(0)} = \frac{(E[A_2] - B_2\xi_0)^2}{2B_2}. \tag{25}$$

Note that there is an important difference between the 2×2 matrix appearing in (24) and the one in (20), since now R_i, R_j are functions of Q_1, Q_2, not independent of them as in (20). We need to determine the way that this dependence affects the various derivatives.

The follower's optimality conditions imply that the system (21) holds. We will rewrite this in terms of $E[q_1]$, $E[q_2]$ and the fundamental parameters of the problem making use of the following identities:

$$E[u] = (B_1 E[q_1] + B_1 E[q_2] + E[BA_1 - B_1 A])/B$$

and

$$\begin{pmatrix} E[p - c_1'(q_1)] \\ E[p - c_2'(q_2)] \end{pmatrix} = -\frac{1}{B} \begin{pmatrix} 1 + BD_1 & 1 \\ 1 & 1 + BD_2 \end{pmatrix} \begin{pmatrix} E[q_1] \\ E[q_2] \end{pmatrix} + \frac{1}{B} \begin{pmatrix} \overline{A} - BC_1 \\ \overline{A} - BC_2 \end{pmatrix}$$

which is due to the identity $E[p] = E[A] - B(E[q_1] + E[q_2])$. So (21) becomes

$$\Gamma \begin{pmatrix} E[q_1] \\ E[q_2] \end{pmatrix} = \Psi \begin{pmatrix} \overline{A} - BC_1 \\ \overline{A} - BC_2 \end{pmatrix} - (BQ_1 + BQ_2 - E[BA_1 - B_1 A]) \begin{pmatrix} -\xi_1 \\ -\xi_2 \end{pmatrix}, \qquad (26)$$

where we have written

$$\Psi = \begin{pmatrix} \psi_{11} & -\psi_{21} \\ -\psi_{12} & \psi_{22} \end{pmatrix}, \quad \Gamma = \Psi \begin{pmatrix} 1 + BD_1 & 1 \\ 1 & 1 + BD_2 \end{pmatrix} + B_1 \begin{pmatrix} \xi_1 & \xi_1 \\ \xi_2 & \xi_2 \end{pmatrix}.$$

Finally we can calculate the derivatives we require: taking derivatives of (26) with respect to Q_1 leads to

$$\Gamma \begin{pmatrix} \frac{\partial q_1}{\partial Q_1} \\ \frac{\partial q_2}{\partial Q_1} \end{pmatrix} = \begin{pmatrix} B\xi_1 \\ B\xi_2 \end{pmatrix}.$$

Now Γ is invertible because

$$det(\Gamma) = (\psi_{11}\psi_{22} - \psi_{21}\psi_{12})(B^2 D_1 D_2 + BD_1 + BD_2)$$
$$+ BB_1(\psi_{22}D_2\xi_1 + \psi_{11}D_1\xi_2 + \psi_{12}D_1\xi_1 + \psi_{21}D_2\xi_2) > 0$$

due to $det(\Psi) = \psi_{11}\psi_{22} - \psi_{21}\psi_{12} > 0$ and the fact that all other terms are strictly positive. In fact, Γ is positive definite.

Let $\delta = Q_1 + Q_2 - E[u]$. Then using (21), we have (24) given by

$$\delta \begin{pmatrix} \frac{\partial q_1}{\partial Q_1} & \frac{\partial q_2}{\partial Q_1} \\ \frac{\partial q_1}{\partial Q_2} & \frac{\partial q_2}{\partial Q_2} \end{pmatrix} \cdot \Psi^{-1} \cdot \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

From the first component in this vector equation we can derive

$$\delta \cdot (\xi_1, \xi_2) \cdot (\Gamma^{-1})^T \cdot \Psi^{-1} \cdot \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = 0.$$

Let $\Lambda = \Psi\Gamma^T$, which is the inverse of the 2×2 matrix in the left side of the above equation. We show that Λ is positive definite by checking that both the top left element and the determinant are positive. Now

$$\Lambda_{11} = [-\psi_{21} + \psi_{11}(1 + BD_1) + B_1\xi_1]\psi_{11} - [\psi_{11} - \psi_{21}(1 + BD_2) + B_1\xi_1]\psi_{21}$$
$$= [BD_1(B + \beta_2)\xi_1 + (B + B_1)\xi_1](B + \beta_2)\xi_1 - [(B + B_1 - BD_2\beta_2)\xi_1]\beta_2\xi_1$$
$$= \xi_1^2[BD_1(B + \beta_2)^2 + B(B + B_1) + BD_2\beta_2^2] > 0$$

and $det(\Lambda) = (\psi_{11}\psi_{22} - \psi_{21}\psi_{12})det(\Gamma) > 0$.

Thus Λ^{-1} is positive definite, which implies

$$\delta = Q_1 + Q_2 - E[u] = 0$$

since $(\xi_1, \xi_2) \neq (0, 0)$.

Therefore from (21), we have

$$E[p] = E[c_1'(q_1)] \text{ and } E[p] = E[c_2'(q_2)],$$

that is, the expected spot market price equals the expected marginal cost of generation of the two generators. Moreover, since $E[p - c_i'(q_i)] = (E[q_i] - (Q_i + R_i))/(B + \beta_j)$, we have $E[q_i] = Q_i + R_i$ for $i, j = 1, 2, i \neq j$. That is, generator i is fully contracted. Because $E[u] = Q_1 + Q_2$, we have $E[v] = E[q_1 + q_2 - u] = R_1 + R_2$, that is, retailer 2 (the follower) is also fully contracted.

Since $E[q_i], E[q_j]$ are determined by conditions $E[p] = E[c_1'(q_1)]$ and $E[p] = E[c_2'(q_2)]$ (see (11), (12)), they are independent of how the two retailers offer the contracts; so are the retailers' expected consumptions, total contract levels and expected profits for all the retailers and generators. Therefore f is uniquely determined in (25). ∎

Proposition 7 *The following functions are concave:*

(a) $U(Q_1, Q_2) = E[\frac{A}{B}q - \frac{1}{2B}q^2 - c_i(q_i) - c_j(q_j) - \pi_i^{(0,0)} - \pi_j^{(0,0)}]$

(b) $V(Q_i) = E[\frac{A}{B}q - \frac{1}{2B}q^2 - pq_j - c_i(q_i) + (p - \bar{f}_j)\bar{Q}_j - \pi_i^{(0)}]$

24

$$(c) \quad L(Q_i, Q_j) = \quad E[\tfrac{A_1}{B_1}u - \tfrac{1}{2B_1}u^2 - p\,u + p(q_i + q_j) - c_i(q_i) - c_j(q_j)$$
$$- (p - g_i)R_i - (p - g_j)R_j - \pi_i^{(0,0)} - \pi_j^{(0,0)}]$$

Proof

(a) Using the fact that $\frac{A}{B} - \frac{1}{B}q = p$ we have

$$\frac{\partial U}{\partial Q_i} = E[p](\frac{\partial q_i}{\partial Q_i} + \frac{\partial q_j}{\partial Q_i}) - E[c_i'(q_i)]\frac{\partial q_i}{\partial Q_i} - E[c_j'(q_j)]\frac{\partial q_j}{\partial Q_i}.$$

Thus

$$\frac{\partial^2 U}{\partial Q_i^2} = \frac{\partial p}{\partial Q_i}(\frac{\partial q_i}{\partial Q_i} + \frac{\partial q_j}{\partial Q_i}) - c_i''(q_i)\left(\frac{\partial q_i}{\partial Q_i}\right)^2 - c_j''(q_j)\left(\frac{\partial q_j}{\partial Q_i}\right)^2$$
$$= -\xi_i^2[B + D_i(B + \beta_j)^2 + D_j\beta_j^2] \tag{27}$$

since $\partial q_i/\partial Q_i = \psi_{ii} = (B + \beta_j)\xi_i$ and $\partial q_j/\partial Q_i = -\psi_{ji} = -\beta_j\xi_i$. Similarly

$$\frac{\partial^2 U}{\partial Q_i \partial Q_j} = \frac{\partial p}{\partial Q_j}(\frac{\partial q_i}{\partial Q_i} + \frac{\partial q_j}{\partial Q_i}) - c_i''(q_i)\frac{\partial q_i}{\partial Q_j}\frac{\partial q_i}{\partial Q_i} - c_j''(q_j)\frac{\partial q_j}{\partial Q_j}\frac{\partial q_j}{\partial Q_i}$$
$$= \xi_i\xi_j[-B + D_i(B + \beta_j)\beta_i + D_j(B + \beta_i)\beta_j]$$
$$= \xi_i\xi_j B$$

using the fact that $2B = D_i(B + \beta_j)\beta_i + D_j\beta_j(B + \beta_i)$ by (23). Hence the Hessian of U is

$$H(U) = \begin{pmatrix} -\xi_i^2(B + \gamma_i) & \xi_i\xi_j B \\ \xi_i\xi_j B & -\xi_j^2(B + \gamma_j) \end{pmatrix}$$

where $\gamma_i = D_i(B + \beta_j)^2 + D_j\beta_j^2$ and $\gamma_j = D_j(B + \beta_i)^2 + D_i\beta_i^2$. To establish that this is negative definite we just need to check that $\frac{\partial^2 U}{\partial Q_1^2} < 0$, which follows from (27), and show that the determinant is positive. Now

$$\det(H(U)) = \xi_i^2\xi_j^2((B + \gamma_i)(B + \gamma_j) - B^2) > 0$$

as required. So the Hessian is negative definite and hence the function is concave.

(b) We have

$$\frac{\partial V}{\partial Q_i} = E[(\frac{A}{B} - \frac{1}{B}q)\frac{\partial q}{\partial Q_i} - q_j\frac{\partial p}{\partial Q_i} - p\frac{\partial q_j}{\partial Q_i} + \bar{Q}_j\frac{\partial p}{\partial Q_i} - c_i'(q_i)\frac{\partial q_i}{\partial Q_i}]$$
$$= E[p](\frac{\partial q}{\partial Q_i} - \frac{\partial q_j}{\partial Q_i}) + E[\bar{Q}_j - q_j]\frac{\partial p}{\partial Q_i} - E[c_i'(q_i)]\frac{\partial q_i}{\partial Q_i}$$

So

$$\frac{\partial^2 V}{\partial Q_i^2} = \frac{\partial p}{\partial Q_i}(\frac{\partial q}{\partial Q_i} - \frac{\partial q_j}{\partial Q_i}) - \frac{\partial q_j}{\partial Q_i}\frac{\partial p}{\partial Q_i} - c_i''(q_i)(\frac{\partial q_i}{\partial Q_j})^2$$
$$= \xi_i^2(-(B + \beta_j) - \beta_j - D_i(B + \beta_j)^2) < 0$$

25

as required.

(c) We have

$$
\begin{aligned}
\frac{\partial L}{\partial Q_i} &= \frac{\partial p}{\partial Q_i} E[q_i + q_j - u - R_i - R_j] + E[p\frac{\partial(q_i + q_j)}{\partial Q_i} - c_i'(q_i)\frac{\partial q_i}{\partial Q_i} - c_j'(q_j)\frac{\partial q_j}{\partial Q_i}] \\
&= \frac{\partial p}{\partial Q_i} E[q_i + q_j - u - R_i - R_j] + \frac{\partial U}{\partial Q_i}
\end{aligned}
$$

Hence

$$
\begin{aligned}
\frac{\partial^2 L}{\partial Q_i^2} &= \frac{\partial p}{\partial Q_i}(\frac{\partial q_i}{\partial Q_i} + \frac{\partial q_j}{\partial Q_i} + B_1\frac{\partial p}{\partial Q_i}) + \frac{\partial^2 U}{\partial Q_i^2} \\
&= \xi_i^2(-B + B_1) + \frac{\partial^2 U}{\partial Q_i^2}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial^2 L}{\partial Q_i \partial Q_j} &= \frac{\partial p}{\partial Q_i}(\frac{\partial q_i}{\partial Q_j} + \frac{\partial q_j}{\partial Q_j} + B_1\frac{\partial p}{\partial Q_j}) + \frac{\partial^2 U}{\partial Q_i \partial Q_j} \\
&= \xi_i\xi_j(-B + B_1) + \frac{\partial^2 U}{\partial Q_i \partial Q_j}
\end{aligned}
$$

Thus the Hessian of L, is

$$
H(L) = (B_1 - B) \begin{pmatrix} \xi_i^2 & \xi_i\xi_j \\ \xi_i\xi_j & \xi_j^2 \end{pmatrix} + H(U)
$$

which is the sum of a negative semi-definite matrix and a negative definite matrix (from (a)), and hence is negative definite.

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